Topic: Logarithmic differentiation

Question: Use logarithmic differentiation to find the derivative.

$$y = 3^{5x}$$

Answer choices:

A $y' = 15^{5x}\left(\ln 3\right)$

B $y' = 3^{4x}\left(5 \ln 3\right)$

C $y' = 3^{5x}\left(5 \ln 3\right)$

D $y' = 3^{5x}\left(\ln 15\right)$

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Solution: C

To use logarithmic differentiation to find the derivative, we'll take the natural log of both sides.

$$y = 3^{5x}$$

$$\ln y = \ln\left(3^{5x}\right)$$

When you have an exponent in your log function like this, you can move it out to the front.

$$\ln y = 5x \ln 3$$

Then take the derivative of both sides. You'll need implicit differentiation on the left, since y isn't just a variable, it's a function, and you'll need product rule on the right side.

$$\frac{1}{y}(y') = (5)(\ln 3) + (5x)(0)$$

$$\frac{1}{y}(y') = 5 \ln 3$$

$$y' = 3^{5x}\left(5 \ln 3\right)$$

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Topic: Logarithmic differentiation

Question: Find the derivative.

$$f(x) = 3\ln(5x)$$

Answer choices:

A $f'(x) = \frac{5}{x}$

B $f'(x) = \frac{3}{x}$

C $f'(x) = \frac{3}{5x}$

D $f'(x) = \frac{15}{x}$

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Solution: B

The derivative of $\ln x$ is given by

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

So the derivative of the given function

$$f(x) = 3\ln(5x)$$

is

$$f'(x) = (3)\frac{1}{5x}(5)$$

$$f'(x) = \frac{15}{5x}$$

$$f'(x) = \frac{3}{x}$$

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Topic: Logarithmic differentiation

Question: Differentiate the function.

$$f(x) = \ln\left(6x^2 - 3x + 9\right)^{4x}$$

Answer choices:

A $f'(x) = 4\ln\left(6x^2 - 3x + 9\right)$

B $f'(x) = \dfrac{4x(4x-1)}{2x^2 - x + 3}$

C $f'(x) = 4\ln\left(6x^2 - 3x + 9\right) + \dfrac{4x(4x-1)}{2x^2 - x + 3}$

D $f'(x) = 4\ln\left(6x^2 - 3x + 9\right) + \dfrac{1}{6x^2 - 3x + 9}$

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Solution: C

When you've got a power function inside the log function, you can move the exponent out in front of the logarithm.

$$f(x) = \ln\left(6x^2 - 3x + 9\right)^{4x}$$

$$f(x) = 4x \ln\left(6x^2 - 3x + 9\right)$$

Now we'll need to use product rule to take the derivative. The derivative of $\ln x$ is given by

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

So the derivative of

$$f(x) = 4x \ln\left(6x^2 - 3x + 9\right)$$

is

$$f'(x) = (4)\left[\ln\left(6x^2 - 3x + 9\right)\right] + (4x)\left(\frac{1}{6x^2 - 3x + 9}\right)(12x - 3)$$

$$f'(x) = 4 \ln\left(6x^2 - 3x + 9\right) + \frac{48x^2 - 12x}{6x^2 - 3x + 9}$$

$$f'(x) = 4 \ln\left(6x^2 - 3x + 9\right) + \frac{12x(4x - 1)}{3(2x^2 - x + 3)}$$

$$f'(x) = 4 \ln\left(6x^2 - 3x + 9\right) + \frac{4x(4x - 1)}{2x^2 - x + 3}$$

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